New-Math Influences on Selective Entrance Examinations into High Schools In Iceland

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Introduction

- An entrance examination into high schools in Iceland was established in 1946, intended to provide equal opportunities for education.
- By the mid-1960s it became considered a hindrance on young people's path to prepare for life.
- In the midst of increased demands for education for all, New Math was implemented, expected to facilitate understanding.
- The presentation contains an analysis and comparison of typical examination papers before and after the implementation period of New Math during 1966–1968.

Questions

- The questions that arise concern educational expectations that can be gleaned from the examination papers:
  - what content and performance expectations were considered optimal preparation for further studies?
  - what changes did the implementation of the New Math bring?
  - did the changes promote better understanding?

Background - History

- Iceland belonged to the **Danish realm from late 14th century until 1944**
- When Danish learned schools were split into language-history and mathematics-physics streams in 1877, Icelanders chose a language-history stream for their sole learned school.
- Euclidean geometry was restored in a mathematics-physics stream established in 1919.
- The Danish school system was **split into a lower and upper secondary level in 1903** while this was not **the case in Iceland until 1946.**
- In the early 1930s, educational opportunities in Iceland consisted of **two six year schools with own selective entrance examinations,** several technical schools, and lower secondary schools in towns and rural areas, providing general education such as arithmetic and languages, but **without pathways to the higher education.**
- The upper level will from now on be called **high school.**

The national entrance examination

- In 1946, the new-born Republic of Iceland issued a new education act (Law no. 22/1946), creating a uniform educational system with **eight-year compulsory education** and **equal access to high school** education.
- A **national high school entrance examination** in eight main subjects was run during 1946–1976 in lower secondary schools all around the country.
- The high-school authorities were dissatisfied that their **former six-year program was reduced to four years**, and that they were **deprived of selecting their students.**
- As a compromise, regulations (no. 3/1937) for the former six-year schools' second year were chosen as a basis for that official examination.

Goals of the national exmination

- No goals of the national examination were stated in 1946 while in 1961 they were analyzed as:
  - to ensure a certain and standardized **minimum knowledge** in a number of subjects;
  - the **selection of the fittest** with respect to certain attributes, considered necessary for those who wanted to study in a high school and at a university or other higher education;
  - to offer all students and their relatives a certain assurance of an **impartial assessment**; and identical examinations for all students.

Problems of the entrance examination

- Increased wish for **education for all**
  - Until 1960, a constant rate of 20% of the cohort attempted the examination and 13-14% reached high-school-admission minimum grade.
  - By 1969 the rates had risen to 34% vs. 21% and more in the 1970s.
- From 1966 measures were taken to make the test easier to grade and more accessible for average students:
  - the **examination time was shortened**,
  - **seen problems,** reviewed exercises from class, **were replaced by small problems**, testing one item each, presumably to help the less able students to show basic competences,
  - the **number of test items rose** from 6 – 8 to 50 and later 100 small and often unrelated items.

Critique of the entrance examination

- Psychologist Professor M. Jónasson (1968):
  - the examination had for a long time **had the role of filtering or selecting**, which was neither painless nor infallible.
  - This could be justified in nations with educational institutions in a constant funding crisis, where **channelling only the fittest students into higher education** seemed the preferable utilization of available educational provisions.
  - However, **preparation time was far too short**, teachers needed more time to learn to know the capacity and the diligence of their students and to give them guidance.
  - One year only led to too tight a time schedule, pressure and hurried work which a **youngster in a formative period could not easily sustain.**

Critique of the entrance examination

- Psychologist Professor M. Jónasson (1968), cont.:
  - The **host of incoherent details** that the students were expected to remember was horrifying.
  - Would the answers to such questions be the **correct measure of the capacity of youngsters** for higher education?
  - What about **inventiveness, judgement, reasoning and creativity**?
- However, the mathematics examination began to develop from 1966 even more into incoherent details.

Before New Math: Seen and unseen problems

- Until 1966 the mathematics examination was divided **into seen problems and unseen problems**, with equal weight, tested two days in a row.
- Students had solved the seen problems previously in class with the help of their teacher.
- The unseen part was typically 6–8 problems; 4–6 **story problems on area, volume and proportions, solved by setting up equations**; and two rather complicated **fraction problems with algebraic expressions** in denominators.
- The story problems concerned situations in contemporary daily life, or were versions of old problems, even from Fibonacci's *Liber Abaci*.
- In the first year, 1946, examining all over the country in **Euclidean geometry** as prescribed in the former high-school regulations **proved not to work** as many teachers had not studied it and was dropped after which only measurement represented geometry.

New Math and the entrance examination

- The aim of the New Math expressed in 1968 for the 1969 test was
  - to base school mathematics on the **basic concepts of the set theory**, which simultaneously were simple and general,
  - to increase emphasis on **the meaning and nature of numbers and of number computations.**
- These changes required a different approach in the national examination, where the basis was laid for algebra:
  - **Symbolic language of set theory** allowed ideas and their relations to be expressed in an exact and clear way.
  - It was desirable to **delay the conventional algebra of numbers** until students acquired mastery of the relations of sets and the introduction to set theory.

Results in the examination

- Data from years 1958–1962 indicate that **grades for the seen problems were on average about 12% higher** than the unseen problems.
- During the periods 1952–1955 and 1962–1966, the **mathematics average was always lower than the average of all eight subjects** with an average difference of 5% from the total average of all subjects.
- However, exchanging the seen problems in 1966 for shorter problems did not make a difference in this respect.
- The difference between total average and mathematics average **reduced slightly from 1970**, and in 1972, the national mathematics average was higher than the total average by 2%.

Sub-periods of the entrance examination

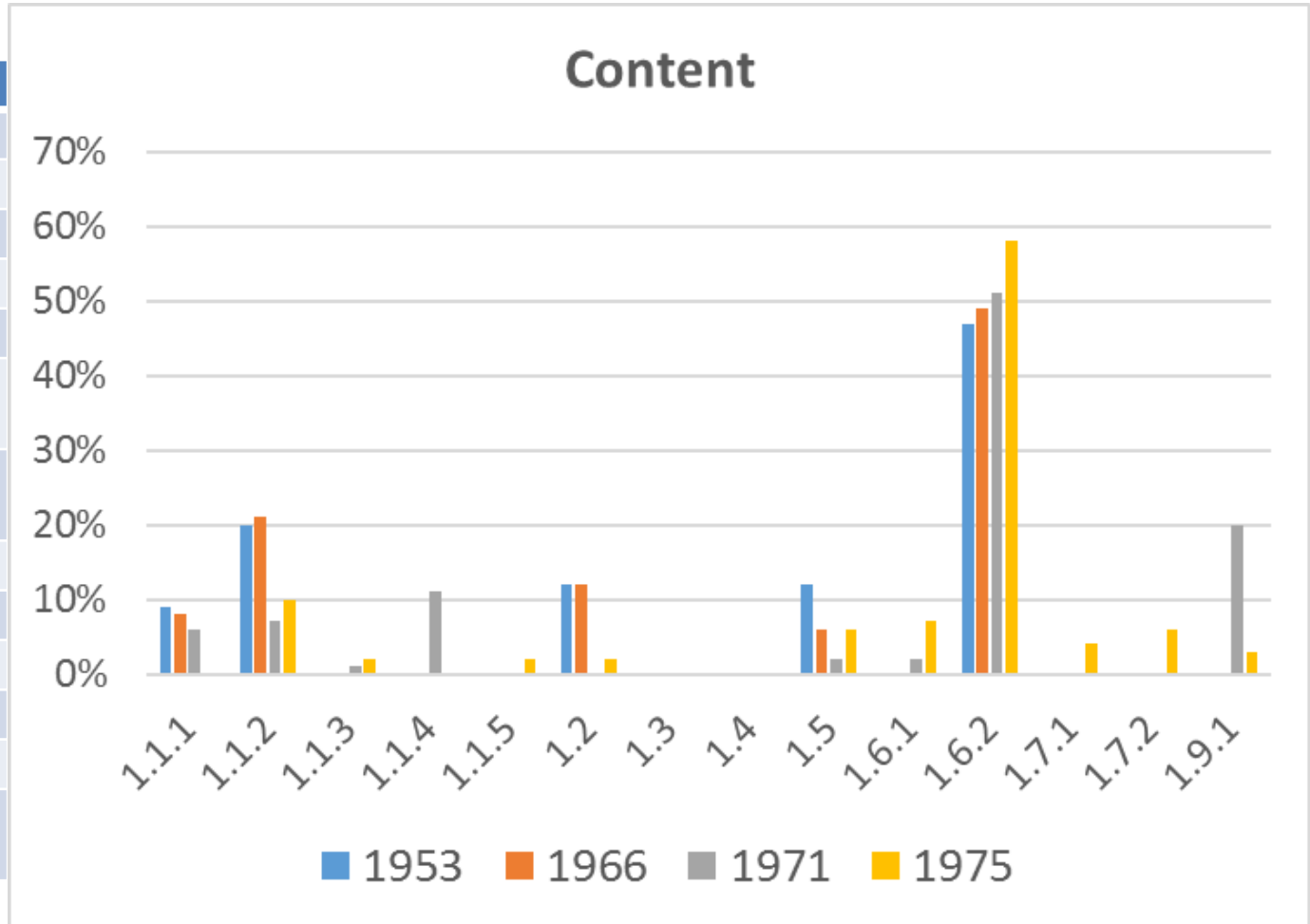
Four examinations papers were chosen for analysis:

- the experimental period 1946–1950
- period of traditional mathematics 1951–1965 **1953**
- transition period 1966–1968 **1966**
- the New Math period 1969–1975
  - 1969–1972 with one syllabus **1971**
  - 1973–1976 two / three syllabi **1975**

Development of content

- Testing fractions reduced –
- Set notation and number concept had only temporary interest –
- Data representation and probability entered –
- Emphasis on algebra increased –
- No geometry –
- Measurement reduced –

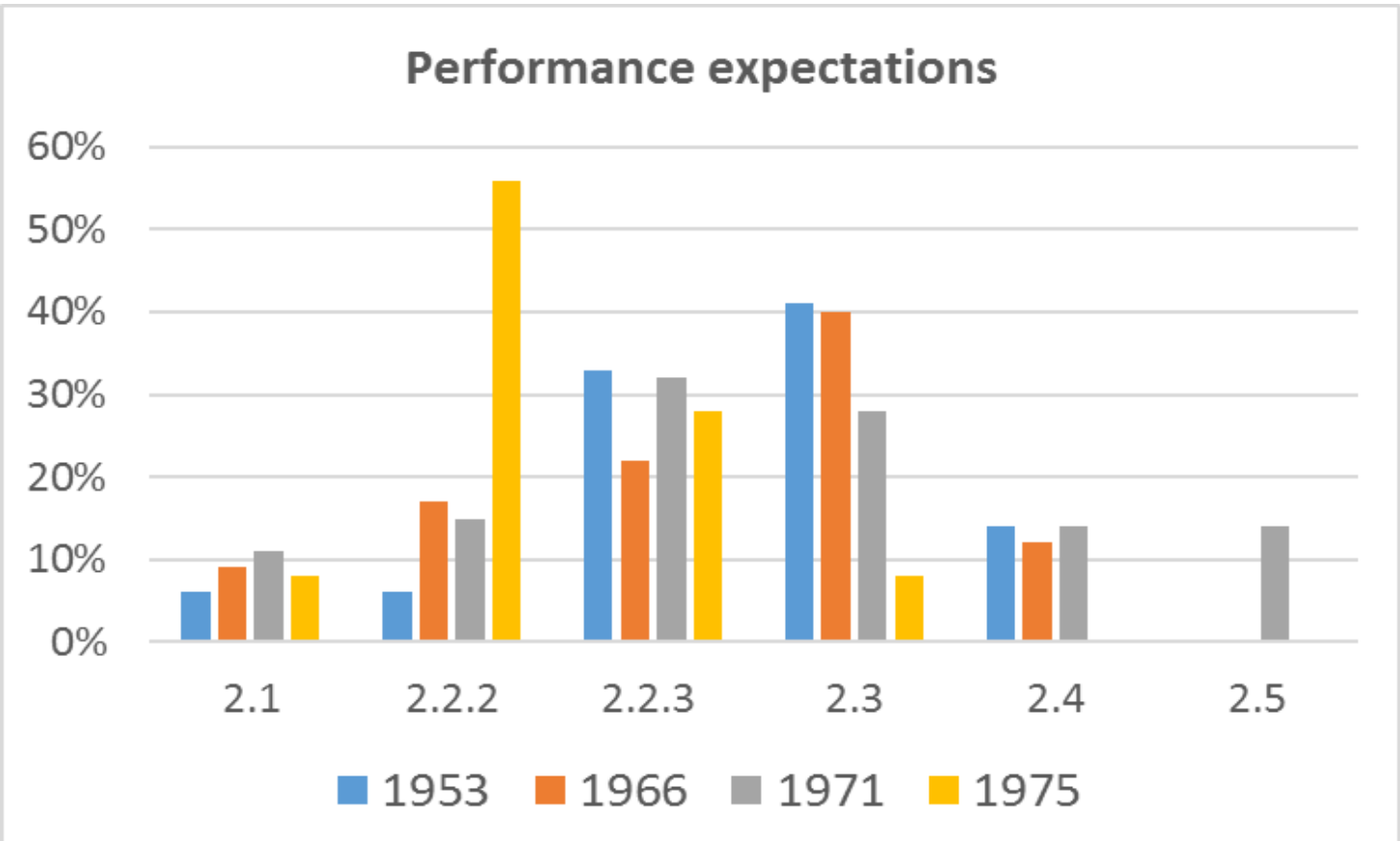
1.1.1	**Whole numbers**
1.1.2	**Fractions and decimals**
1.1.3	**Integer, rationals and real numbers**
1.1.4	**Other numbers and number concepts**
1.1.5	**Estimation and numbers sense**
1.2	**Measurement**
1.3	Geometry: position, visualization and shape
1.4	Geometry: symmetry, congruence, and similarity
1.5	**Proportionality**
1.6.1	**Patterns, relations, and functions**
1.6.2	**Equations and formulas**
1.7.1	**Data representation and analysis**
1.7.2	**Uncertainty and probability**
1.9.1	**Structuring and abstracting (sets, set notation)**



Development of performance expectations

- Performance expectations became less oriented towards independent development of notation, vocabulary, and algorithms
- Students were helped to choose variables in order to be able to form equations out of story problems

2.1	**Knowing**
2.2.2	Performing routine procedures
2.2.3	Using more complex procedures
2.3	Investigation and problem solving
2.4	Mathematical reasoning, developing notation, vocabulary and algorithms
2.5	Communicating, using (set theoretical) vocabulary and notation



Changes in format

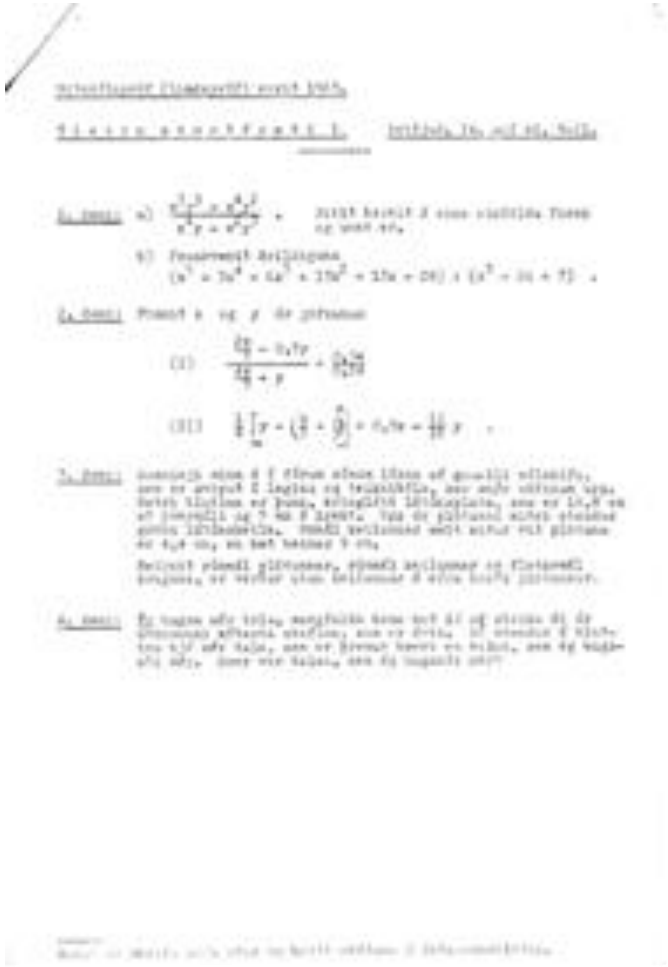


Figure 1. 1963 paper

Figure 2. 1966 paper

Figure 3. 1971 paper

Did implementation of the New Math facilitate understanding?

- The role of set theory in the curriculum seems primarily have been to exercise notation in order to prepare the students for further studies rather than facilitate understanding.
- At this point it could only be used for minimum problem solving, there was not time in one academic year to postpone the introduction of algebra of numbers until the students had acquired mastery of the relations of sets as was proposed in the curriculum document of 1968.
- The role of set theory to increase clarity and facilitate understanding was not relevant as yet.

Theories of understanding

- Skemp distinguished between *instrumental understanding* vs. *relational understanding* –
  - **Instrumental understanding** concerned knowing particular items without relating to previous knowledge
  - In **relational understanding** new concepts relate to a network of ideas and previous knowledge
- Difficulties in emphasizing relational mathematics and relational understanding lied e.g. in the **backwash effect of examination**, **overburdened syllabi** and difficulty in **assessment** of whether a person understands relationally or instrumentally.
- Skemp suspected that much of what was being taught under the description of New Math, was being taught and learnt just as instrumentally as were the syllabi which have been replaced
  - possibly due to mismatch between teachers whose conception of understanding is instrumental and aims implicit in the content.

Theories of understanding

- Anna Sierpinska: distinguished between *acts of understanding* and *processes of understanding* as lattices of acts of understanding, linked by reasonings (explanations, validations).
- A relatively good understanding could be achieved if the process of understanding contained a certain number of especially significant acts, namely acts of overcoming obstacles specific to that mathematical situation.
- George Polya: defined four-step problem solving procedure: Understanding – Devising a plan – Carrying out the plan – Looking back.
- Understanding consisted of realizing what was unknown, which data were available, and what was the condition.

Remaining questions

- Do the following issues enhance mathematical thinking?
  - Host of incoherent details of diffused focus.
  - Performance expectations less oriented towards independent development of notation, vocabulary, and algorithms.
- Polya suggested to think of a familiar problem in devising a plan – Did the seen problems enhance understanding?
- Skemp: does backwash effect of examination, and overburdened syllabi promote the more superficial *instrumental understanding* at the cost of *relational understanding*?
- Sierpinska's vocabulary: Could long story problems from textbooks and previous examination papers provide opportunity for teachers to delve deeply into composite problems together with their students and even create by them a *lattice of acts of understanding*?

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Analysis of examination papers

- The analysis is based on the curriculum framework for TIMSS by Robitaille, Mc Knight, Schmidt, Britton, Raizen and Nicol (1993).
- Papers chosen:
  - **1953**, when the examination had become established with **traditional mathematics** – 6 large problems of unseen problems.
  - **1966**, **right before the implementation of the New Math** when the number of participants had grown considerably, and **seen problems removed** – 25 items counted in 18 problems.
  - **1971** when the implementation of the **New Math had become established** – 50 items counted in 36 problems.
  - **1975**, the final year, one of three test versions of New Math 100 items in 37 problems.