Recommendations of the Royaumont Seminar on primary school arithmetic. Influences in the Nordic countries

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Abstract

Following the seminar on new thinking in school mathematics, held in Royaumont, France, in 1959, the Nordic countries took up cooperation on analysing the situation in mathematics education, to work out curriculum plans and write experimental texts. A Danish author, A. Bundgaard, and her collaborator wrote a textbook series for primary level which was translated into Icelandic. The text is analysed with respect to presentations on arithmetic education at the seminar and compared to previous and later texts in use. The results show that the declared intention of the reform movement to emphasize the structure of the number system and build its presentation on set-theoretical concepts gradually faded out while the study of numbers built on primes and divisibility became a revived topic in Icelandic school mathematics. Furthermore new topics such as statistics and introduction to probability entered the curriculum.

Introduction

In 1959, a seminar for mathematicians, mathematics educators and mathematics teachers was organized by the OEEC at Royaumont, France, to discuss a reform of school mathematics. Radical reforms were proposed, for arithmetic and algebra teaching, even at primary level. We shall explore

- which ideas on arithmetic teaching, proposed at the Royaumont seminar, were implemented in a primary level textbook series, composed by the Nordic Commission for Modernizing Mathematics [*Nordiska kommittén för modernisering av matematikundervisningen*], abbreviated NKMM
- if the ideas were new in Iceland
- if they survived the first wave of enthusiasm for the New Math and became permanent contribution to school mathematics.

The NKMM primary level textbook series was translated into Icelandic. Their content will be analysed with respect to presentations and recommendations of the Royaumont Seminar, and compared to earlier and later textbook publications in order to clarify permanent influences of the seminar.

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The Royaumont Seminar

The Royaumont Seminar was a seminar on new thinking in school mathematics, held in Royaumont, France, in November 1959. It was arranged by the OEEC (later OECD) and attended by all member countries except Portugal, Spain, and Iceland (OEEC, 1961, pp. 213-220). At the seminar, the European proponents for reform met representatives of the New Math movement in the United States. For a more detailed account of the seminar, see De Bock and Vanpaemel (2015).

During the 1959s and 1960s, CIEAEM, the International Commission for the Study and Improvement of Mathematics Teaching, was an arena in Europe with the aim of finding new approaches to mathematics education suitable to the changed mathematical and social context. Among its founding members were the Swiss psychologist Jean Piaget (1896-1980), mathematicians Gustave Choquet (1915-2006) and Jean Dieudonné (1906-1992) from France. The main concern of the CIEAEM was a growing attention to the student and the process of teaching (CIEAEM website).

The CIEAEM had strong representation at the seminar, with Dieudonné and Choquet among the invited speakers. Choquet introduced the theme of new thinking in mathematics education, and proposed new approaches to arithmetic and algebra. According to the programme of the seminar, the Danish mathematician Svend Bundgaard spoke on teachers' mathematical competencies and their training on its second last day (Schubring, personal communication).

Dieudonné belonged to the Bourbaki group, a group of mathematicians (mainly French), who worked at a mathematical encyclopedia, where the borders between the different mathematical topics were abolished. The group's central concept was "structure". When describing the structures, the importance lay in the elements' relationships, determined by axioms.

Among speakers on behalf of the reform movement in the United States were Howard Fehr, head of department of teaching mathematics at Teacher College of Columbia University in New York, and Marshall Stone, chairman of ICMI, the Commission of Mathematics Instruction of the International Union of Mathematics. Stone was the chairman of the Royaumont seminar.

Stone emphasized the need for reform to give quite as much attention in schools to the technical preparation of skilled workers as to the training of future university students, engineers and scientists (OEEC, 1961, p. 19), thus promoting education for all. He called for a thorough up-to-date analysis of simple uses of the more elementary kinds of mathematics in the skilled occupations of modern industry and in the daily life of the average citizen called on to vote and pay taxes. Stone expressed his concern that elementary mathematics must not repel the child. It was all too evident that primary schools were failing to develop adequately the latent mathematical talents and interest of the average child. It was imperative to find remedies for these defects in elementary mathematical instruction. Fortunately, illuminating psychological investigations, particularly those of Piaget, were pointing the way to hitherto unrecognized pedagogical possibilities (OEEC, 1961, p. 22-23). However, Stone's plea for research and for reforming primary teaching, remained without success, and his programme of "mathematics for all" was not endorsed in the follow-up discussion nor the final report of the meeting (Schubring, 2014b).

During the last two days of the seminar, the participants developed jointly its conclusions. The conclusions, published in a report (OEEC, 1961) by Howard Fehr, are not identical to the conclusions preserved in the archives of OECD. However, the original conclusions of the seminar concerning arithmetic teaching (OEEC, 1961, pp. 108-111; Schubring, 2014a, pp. 93-94) do not differ considerably with respect to the topics discussed in this article. In the following, we shall explore the official report with respect to recommendations on arithmetic and algebra teaching and how they are reflected in a Nordic primary level textbook series.

Nordic cooperation

One of the final recommendations of the Royaumont Seminar was that each country could reform its mathematics teaching according to its own needs, but it was recommended to establish as much cooperation as possible. The Nordic participants in the Royaumont Seminar agreed upon cooperation on reform of mathematics teaching. The ideas about Nordic cooperation were presented to governmental bodies, and the issue was taken up in the Nordic Council, which decided to set up a committee under its Culture Commission. Each of four countries – Denmark, Finland, Norway and Sweden – appointed four persons to the Nordic Committee for Modernizing Mathematics Teaching, NKMM.

The committee was active from 1960 until 1967, when its report was ready in the autumn. The members of the committee were mathematicians and mathematics teachers, or they came from school administration. Their programme for reform was to

- analyse the situation in mathematics education
- work out preliminary and revised curriculum plans, and
- write experimental texts for courses at all school levels.

The committee appointed several teams of writers. Its focus was on the mathematical content, and the teaching of seventh to twelfth grades was its main object. Consequently its main contribution was in this field. However it was decided to handle mathematics courses throughout the school and the committee contacted for that purpose extra experts for the first to sixth grades.

Writing sessions were arranged in summer 1961. Some texts were ready that autumn, and the others were to be so successively until the beginning of 1966. Joint Nordic manuscripts were planned. Several persons from each country would translate and adapt the joint publications to each language. This committee dominated mathematics instruction in the Nordic countries for most of the 1960s (Gjone, 1983, II. pp. 78-80; Nordisk råd, 1967).

Denmark was one of the countries which went the furthest when it came to introducing the Bourbaki tradition into university programs, and eventually also to high school programs. Svend Bundgaard was a highly influential professor of mathematics at Aarhus University, and when he took up this professorship around 1954, after spending time abroad, he declared: "This New Math is something we must do in Denmark. We have to really revamp the entire program and modernize it." He was also one of those who arranged for Danish participation at the Royaumont seminar in France in 1959, and one of its guest speakers. That movement became very influential in Denmark (Karp, 2015).

Svend Bundgaard's sister, primary teacher Agnete Bundgaard, was a member of the writing team. She wrote a textbook series for the first two primary grades together with the Finnish Eeva Kyttä and alone for the remaining four primary grades. Iceland did not participate in the NKMM cooperation. Iceland was part of the Danish Realm since the Kalmar Union around year 1400 until 1944 and many students received their vocational or academic training there still in the 1960s. Icelanders were informed about the textbook series written by Agnete Bundgaard by Svend Bundgaard who had been a co-student of G. Arnlaugsson, an Icelandic mathematician. The series was translated into Icelandic, gradually as it was published in Denmark from 1966 (Bjarnadóttir, 2007, pp. 267-268).

Analysis of the primary level Bundgaard-textbook series

Three arithmetic textbook series were legalized in 1929 for use at primary level in Iceland. Among them were a series by Sigurbjörn Á. Gíslason (1911-1914), here called SÁG, and another by Elías Bjarnason (1927-1929), EB. The EB series was chosen for free distribution in 1939 and was thereafter the only textbook series in use until 1966.

Translation of the NKMM primary level mathematics textbook series for age 7-12 by Agnete Bundgaard and Eeva Kyttä (1967-1968; Bundgaard, 1969-1971) was pursued during 1966-1971. Jens Høyrup (1979) deemed that material as a most orthodox adjustment to the demands of the mathematicians at Royaumont. The series, here called Bundgaard series, caused some disturbance among teachers and parents.

A new series SFG, *Stærðfræði handa grunnskólum 1A … 6B* (Bjarnadóttir et al., 1971-1977), was composed quickly on behalf of the state monopolistic enterprise, State Textbook Publishing House (Ríkisútgáfa námsbóka, RN) and run as the main syllabus for about three decades. By the turn of the century, a new series, GP (Mogensen and Balzer Petersen, 1999-2001; Pálsdóttir et al., 2002-2004) was initiated on behalf of the National Centre for Educational Material (NCEM), the heir of RN, and was run as the main option until 2010.

We shall now compare the two older series, SG and EB, and two more recent series, SFG and GP, to the Bundgaard series with respect to the topics mentioned in Choquet's presentation and the recommendations of the Royaumont seminar. The analysis of the influences splits into three parts; the introduction of

- set theoretical concepts and notation
- structure of the number field, and
- study of numbers.

Proposals on arithmetic teaching realized in Bundgaard series

The second section of the seminar programme was on *new thinking in mathematics education*. The task of the section was to seek answers to what mathematics should be taught, to whom and how. Introducing these and other problems, Gustave Choquet considered the psychological implications of teaching mathematics as well as the presentation of the subject matter. The start of Choquet's address consisted of an exposition of the experiments carried out by Jean Piaget on the understanding of number and magnitude by children up to the age of seven years (OEEC, 1961, p. 62-63). Piaget said that the inclusion of a part in a whole implied a preliminary algebraic structure.

Choquet then spoke on tendencies in modern mathematics: to do away with boundaries between arithmetic, algebra, geometry and calculus, which could be done through the study of structures. The sets of N and Z were endowed with numerous structures, and the set Z constituted an excellent basis for study in that it might be regarded as taking concrete form in the child's mind very early. Its "discrete" character made it tangible so that it might be used for introducing and studying such concepts as one-to-one correspondence, function, conversion and equivalence (OEEC, pp. 63-64). These topics are reflected in the Bundgaard series; see Figures 1, 2 and 3.

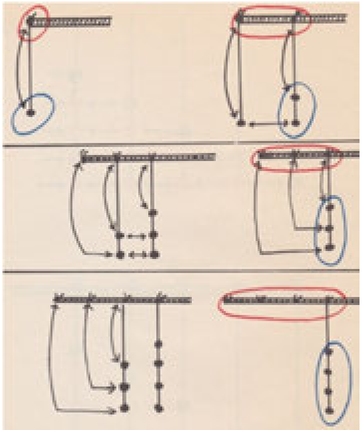


Figure 1. Building up the number concept by one-to-one correspondence to the number line, represented by knots. Age 7 (Bundgaard and Kyttä, 1967, p. 1:20).

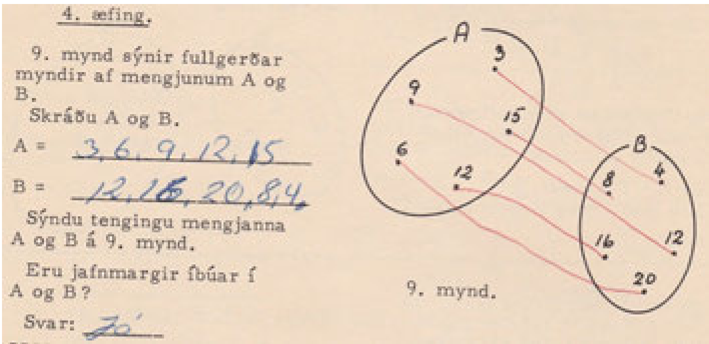


Figure 2. Establishing equivalence between two sets. Age 10 (Bundgaard, 1970, p. 4a:24).

Figure 3. The function concept, relating liters of oil to mass, measured in kg. Age 11 (Bundgaard, 1971, p. 5b:13).

Concerning arithmetic in elementary schools, finite cardinal and ordinal number could be shown by Cuisenaire rods – and concomitant with this, the concepts of the subset of a set, of complementary set, union and intersection of two or three sets could be shown, according to Choquet. The concept of order could be studied from simple examples. See Figure 4 for the reflection in the Bundgaard material. Here, one may notice nurturing of Piaget's idea when he said that the inclusion of a part in a whole implied a preliminary algebraic structure.

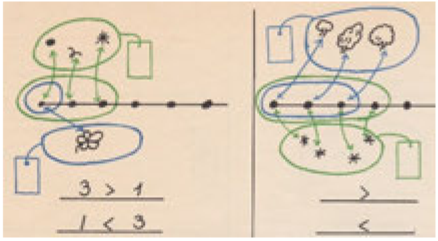


Figure 4. Subsets and ordering. Age 7 (Bundgaard and Kyttä, 1967, p. 1:29).

Addition and multiplication were to be introduced by the union of finite disjoint sets and the product of finite sets, respectively, see Figure 5 for addition and Figure 6 for multiplication in the Bundgaard series.

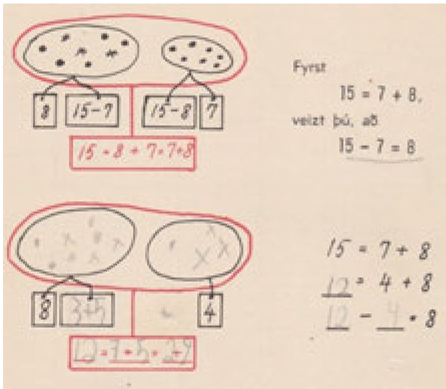


Figure 5. Addition and subtraction. The reader may notice that the pupil, working on the task, was somewhat confused when filling in the empty spaces. Age 8 (Bundgaard and Kyttä, 1968, p. 2a:20).

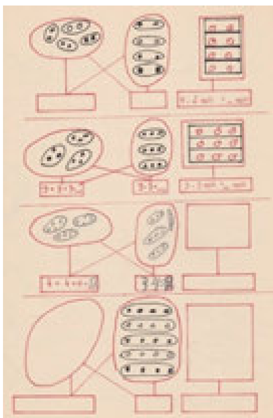


Figure 6. Multiplication, counting apples. The pupil knew too well that 3·4 = 4·3. Age 8 (Bundgaard and Kyttä, 1968, p. 2a:34).

Below, in Table 4, topics related to set theory are collected. These were completely new in the sense that they had not appeared in earlier textbooks. Remnants were seen in later texts, but not as systematically used to build up the number concept. In the following tables, the numbers indicate the age level when the topic in question was introduced. Age within parenthesis indicates that the topic was only marginally introduced and not worked on in what followed. Normally, the topics were readdressed regularly after they had been introduced.

Table 1. Set-theoretical concepts in five Icelandic textbook series for primary level.

Textbook / Topic	SÁG	EB	Bundgaard	SFG	GP
Years in use	1911-37	1927-80	**1966-80**	1971-00	1999-10
Age level	10-13	10-12	**7-12**	7-12	6-12
Sets			**7**	7	10
1 to 1 correspondence			**7**	(7)	
Subset			**7**	7	
Union			**7–10**		11
Intersection			**10**		11
Introduction to set algebra			**10**		
Set difference			**10**		
Complementary set			**10**		
Notation, mod. symbolism			**10**	(11)	
Cuisenaire rods				7	

According to Choquet, the introduction of positive and negative integers raised no difficulty, using translation operators to the right or left. This could be helped by games, e.g. of winning and losing. This would give a better understanding of zero and allow the introduction of simple algebra.

Decimal numeration must be introduced fairly early, confined to a useful system for the notation of large numbers and not for a study of properties of operations. Long multiplications and divisions were unnecessary burdens while children must know simple and rapid mental calculations, and exercise estimation of large numbers.

Fractions could not be avoided but at elementary stage one could not consider p/q as a number but as an operator, operating on magnitudes, e.g. finding 2/3 of a quantity. – Later, when the set of real numbers were introduced as an Archimedean ordered commutative field, the question of fractions would no longer arise since, by definition, p/q would be an element of the field (OEEC, pp. 64-66).

The axioms of the number field were introduced step by step in the Bundgaard series, beginning with the commutative law of addition in its first volume at age 7, see Figure 7, after having presented ordering, see Figure 4.

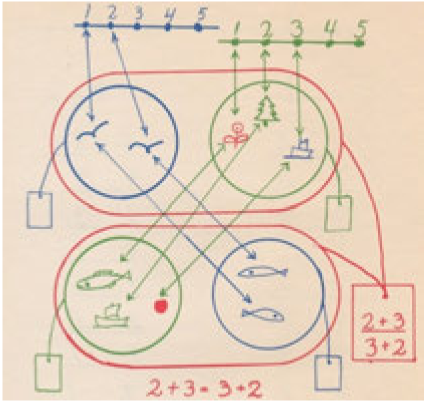


Figure 7. The commutative law. The first introduction of addition and the "+" symbol. Age 7 (Bundgaard and Kyttä, 1967, p. 1:32).

The commutative law was followed by the associative law for addition later in first grade, see Figure 8. Figure 9, demonstrating the distributive law, is the only illustration of people or daily life allowed in the textbook series. The author, Agnete Bundgaard, clearly stressed that she did not want the pupils' minds be distracted by illustrations unrelated to the topics presented. In a letter to Icelandic teachers she said: "Dear Colleagues. It is you who shall try to show the children that the subject <u>in itself</u> is fun and for that aim one can surely only use items that are <u>relevant for the subject"</u> (Bundgaard, 1968, a letter attached to a handbook for teachers). The textbooks for the 7- and 8-year olds were printed in colours, but later textbooks only in black printing.

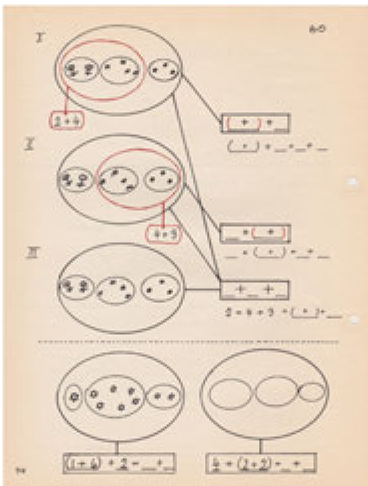
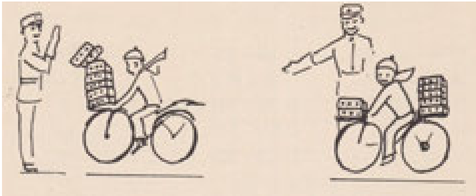
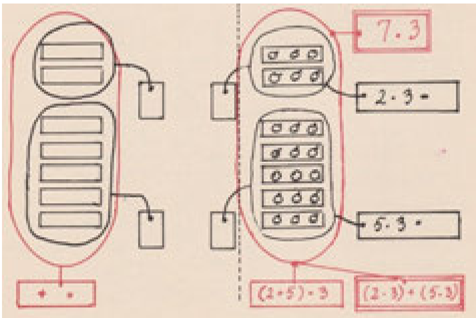


Figure 8. Associative law for addition. Age 7 (Bundgaard and Kyttä, 1967, p. 1:74).





Figures 9 and 10. Distributive law. Age 8 (Bundgaard and Kyttä, 1968, p. 2b:72).

The axioms of the number field were systematically introduced as indicated in Table 2. The commutative and associative laws for addition and multiplication were presented each after other at the age of 7, clearly expressing their names. The additive and multiplicative identities were presented at age 9. The multiplicative inverse was presented in connection with division of fractions at age 12. The additive inverse was not presented as negative numbers were not included in the series.

Table 2. Axioms of the number field in the Bundgaard series compared to other series.

Textbook / Topic	SÁG	EB	Bundgaard	SFG	GP
Years in use	1911-37	1927-80	1966-80	1971-00	1999-10
Age level	10-13	10-12	7-12	7-12	6-12
The commutative law					
of addition			7	7	11
of multiplication	(10)		8	10	11
The distributive law			8	10	11
The associative law					
of addition			7		11
of multiplication			9	12	11
Identity - additive			9		12
- multiplicative			9		12
Inverse - additive				12	
- multiplicative			12		
0 in multiplication 0 in division	(11)		8 10		
Negative numbers				11	11
Inverse operations					
- addition –subtraction					11
- Multiplication – div.					11

In the section of the seminar report, Case for reform – A summary, psychological implications of learning procedures used in primary schools and the shift of school aims to developing concepts and modes of thinking were conceived to demand a corresponding change in arithmetic instruction, probably with the use of some kind of physical objects. Learning must be the result of understanding arising from guided experimentation and discovery. In this way, the child must be led to the abstraction of the quality of a set called its number. In getting to this abstraction it was necessary to use the ideas – but not necessarily the language – of sets, subsets, correspondence and order. A necessary part of the early instruction was the understanding and use of the decimal place number system of numeration. Brighter children could be introduced to the study of number relations involving odd and even numbers, primes, factorization, greatest common factor, least common multiple and place-numeration systems other than ten. There were also areas of disagreement such as the introduction at an early year to negative numbers, use of symbols such as 8 + 1, 7 + 2, … as another name for 9 rather than as operations (pp. 108–110).

One can hardly say that the Bundgaard series promoted guided experimentation or discovery. On the contrary, it emphasized the language of sets and set theory, while it also promoted carefully the study of number relations and the decimal place number system, see Table 3.

Table 3. Topics on numbers in the Bundgaard series compared to other series.

Textbook Topic	SÁG	EB	Bundgaard	SFG	GP
Years in use	1911-37	1927-80	1966-80	1971-00	1999-10
Age level	10-13	10-12	7-12	7-12	6-12
Number line			7	7	6
Number relations: even & odd numbers primes	 13 13	11	 8 9	 8 12	 8 12
Factorization, GCF, LCM	13	(12)	10	12	12
Divisibility – the placement system	13	(11)	9		12
Bases other than ten			9		
Modular systems			11		
Symbols as 7 + 2 for 9			8	7	
Variables, as placeholders - as quantities that vary				7	10 11
Equations				10	10
Probability				9	10
Statistics				8	10
Mental arithmetic	10		8	9	6
Approximation, estimation			11	8	9
Use of calculators					10

Table 3 shows that topics on numbers, such as primes and divisibility, were revived in the Bundgaard series from the early 1900s series SÁG. Bases other than ten and modular systems were novelties that awoke attention. They did not appear in later textbook series, whereas approximation and estimation did. The controversial use of symbols as 7 + 2 for 9, enjoyed some attention when used to promote mental arithmetic, such as 13 + 9 = 13 + (7 + 2) = (13 + 7) + 2 = 20 + 2 = 22.

The topic statistical averages was recommended in the original conclusions (Schubring, 2014a, p. 93) while otherwise probability and statistics, which appeared in later primary level textbooks, were recommended for secondary school level in the seminar report (OEEC, 1961, p. 106-107). Simple equations and variables also began to appear later.

Conclusions

The new topics in the Bundgaards series were primarily the use of set theoretical concepts and notation for building up the number concept and understanding of operations through repeated reference to the axioms of the number field, even if negative numbers were missing. The axioms were carefully introduced with respect to structure. One can therefore agree with Høyrup that the Bundgaard series went far towards meeting the mathematicians' demands. In later textbooks these concepts appeared more as aids to calculations than emphasizing structure.
Building up the system of natural numbers from primes and divisibility, and emphasis on mental arithmetic were revived topics that have survived in the school curriculum to this day. Approximation and estimation have also become permanent contribution to school mathematics in Iceland. What did not become permanent was replaced by introduction to statistics and probability, discussed at Royaumont but less recommended for primary level, and the use of variables and solving simple equations.

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