MASWaves – User guide

Version 2
MASWaves Inversion

June 10, 2020 (draft)

Preface/disclaimers

This document provides guidelines to use the MASWaves Inversion module. The software is written in Matlab. The software and the sample data used in the "quick start guide" can be downloaded free of charge at http://masw.hi.is.

The MASWaves software can be used and modified free of charge. The author(s) take no responsibility for the use of the software, and make no guarantees, expressed or implied, about its quality, reliability, or any other characteristic. Users of MASWaves assume sole responsibility its use in any particular application, for any conclusions drawn from the results of its use, and for any actions taken or not taken as a result of analyses performed using this software.

References

Referencing this software and publications related to its development is highly appreciated. Below is a list of publications related to the development of MASWaves.

MASWaves Inversion (MC) module

Ólafsdóttir, E.Á., Erlingsson, S. & Bessason, B. Open-source MASW inversion tool aimed at shear wave velocity profiling for soil site explorations. Manuscript submitted.

Additional references

Ólafsdóttir, E.Á. (2019). *Multichannel Analysis of Surface Waves for Soil Site Characterization*. [Ph.D. dissertation]. Faculty of Civil and Environmental Engineering, University of Iceland, 308 pp. ISBN: 978-9935-9473-9-0.

Ólafsdóttir, E.Á., Erlingsson, S. & Bessason, B. (2018). Tool for analysis of multichannel analysis of surface waves (MASW) field data and evaluation of shear wave velocity profiles of soils. *Canadian Geotechnical Journal*, 55(2), 217–233. doi:10.1139/cgj-2016-0302. [Open access]

Ólafsdóttir, E.Á., Bessason, B. & Erlingsson, S. (2018). Combination of dispersion curves from MASW measurements. *Soil Dynamics and Earthquake Engineering*, 113, 473-487. doi:10.1016/j.soildyn.2018.05.025. [Open access]

Acknowledgements

1. Introduction

MASWaves (Multichannel Analysis of Surface Waves for assessing shear wave velocity profiles of soils) is an open-access MATLAB package, developed at the Faculty of Civil and Environmental Engineering, University of Iceland, for processing and analyzing multichannel surface wave records using the MASW technique.

The MASWaves software consists of three separate modules (MASWaves Dispersion, Combination and Inversion). The function of the dispersion analysis module (MASWaves Dispersion) is to identify experimental dispersion curves (DCs) from multi-channel surface wave records. An experimental curve obtained from a single multi-channel registration is referred to as an 'elementary dispersion curve'. The purpose of the second processing tool (MASWaves Combination) is to combine dispersion curves obtained from multiple surface wave registrations and to assess the uncertainty associated with the composite curve estimate. The inversion tool (MASWaves Inversion) is used to evaluate the shear wave velocity profile of the tested site by inverting the composite fundamental mode dispersion curve. The inversion is conducted using a Monte Carlo (MC) based global search algorithm with the fast delta-matrix algorithm[1] used for computation of theoretical dispersion curves.

This document provides guidance to the use of the MASWaves Inversion module. The module consists of the following .m-files. A description of each .m-file and a list of its input and output arguments is provided in sections 3.1 to 3.6.

- *MASWaves_inversion_MC.m*
- *MASWaves_theoretical_dispersion_curve_FDMA.m*
- *MASWaves_FDMA.m*
- *MASWaves_FDMA_recursion.m*
- *MASWaves_misfit_MC.m*
- *MASWaves_inversion_MC_plot.m*

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[1] Buchen, P.W. & Ben-Hador, R. (1996). Free-mode surface-wave computations. *Geophysical Journal International*, 124(3), 869–887. doi:10.1111/j.1365-246X.1996.tb05642.x.

2. Quick start guide

- Download the MASWaves Inversion module and the sample data (a total of 9 files in two directories). Open MATLAB. Go to the directory "MASWaves_Inversion" and add the different directories of the MASWaves Inversion module on the MATLAB path. The file *Test_MASWaves_Inversion_MC.m* (in the examples directory) contains the code used to run the following example.

- Import and view the sample data (*SampleData_MASWavesInversion.csv*). A dispersion curve with upper and lower boundaries as displayed in Figure 1 should appear.
  - The sample data was recorded at a normally dispersive silty sand site. Twenty-four 4.5 Hz geophones were used as receivers with a receiver spacing of 2 m. The measuring frequency was 1000 Hz. Repeated shots were applied at 3-30 m distance from "receiver 1" (forward measurements) or "receiver 24" (backward measurements). The elementary dispersion curve estimates were subsequently added up within logarithmically spaced wavelength intervals.[2] The upper and lower boundary curves correspond to plus-minus one standard deviation of the mean composite curve.

```matlab
% Load and plot sample data
SampleData = readtable('examples\SampleData_MASWavesInversion.csv');
c_OBS = SampleData{2:end,1};
up_low_boundary = 'yes'; % Upper/lower boundary curves are available
c_OBS_low = SampleData{2:end,2};
c_OBS_up = SampleData{2:end,3};
lambda_OBS = SampleData{2:end,4};

figure, hold on
exp_curve = plot(c_OBS,lambda_OBS,'kx-','MarkerFaceColor',...
    'k','MarkerSize', 2,'LineWidth',1);
exp_low = plot(c_OBS_low,lambda_OBS,'r--','LineWidth',1);
exp_up = plot(c_OBS_up,lambda_OBS,'r--','LineWidth',1);

set(gca,'FontSize',8),axis ij, grid on, box off
xlim([80 200]), ylim([0 35])
xlabel('Rayleigh wave velocity [m/s]','FontSize',8,'Fontweight','normal')
ylabel('Wavelength [m]','FontSize',8,'Fontweight','normal')
legend([exp_curve,exp_low],'mean','mean \pm std','Location','SouthWest')

set(gcf,'units','centimeters')
pos = [2, 2, 6, 8];
set(gcf,'Position',pos)
```

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[2] Ólafsdóttir, E.Á., Bessason, B. & Erlingsson, S. (2018). Combination of dispersion curves from MASW measurements. *Soil Dynamics and Earthquake Engineering*, 113, 473-487. doi:10.1016/j.soildyn.2018.05.025. [Open access]
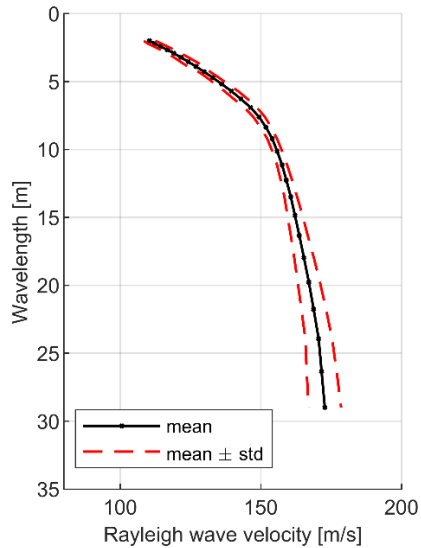
Figure 1. Experimental dispersion curve with upper/lower boundaries (sample data).

- Specify a range for the testing Rayleigh wave phase velocity.

- Specify an initial layer model for the inversion analysis. The parameters required to specify the initial model are number of finite thickness layers (n), layer thicknesses (h), shear wave velocity (β), mass density (ρ) and compressional wave velocity (α). In addition, the number of unsaturated soil layers (n_{unsat}) and the Poisson's ratio of the unsaturated soil are required as input parameters.

```
% Testing phase velocity
c_min = 100; % m/s
c_max = 300; % m/s
c_step = 0.09; % m/s
delta_c = 5; % m/s

% Initial values for model parameters
n = 4;
h_initial = [2 2 4 6]; % m
beta_initial = [c_OBS(1)*1.09 c_OBS(10)*1.09 c_OBS(17)*1.09...
    c_OBS(23)*1.09 c_OBS(end)*1.09]; % m/s
n_unsat = 1;
nu_unsat = 0.3;
alpha_temp = sqrt((2*(1-nu_unsat))/(1-2*nu_unsat))*beta_initial; % m/s
alpha_initial = [alpha_temp(1) 1440 1440 1440 1440]; % m/s
rho = [1850 1900 1950 1950 1950]; % kg/m^3
```

- Specify the number of soil layers where velocity reversals are permitted. Here the value of the input parameter $N_{reversals}$ is specified as '0' for a normally dispersive analysis.

- Specify the values of the search-control parameters b_S, b_h, N_{max} and e_{max} and run the analysis (*MASWaves_inversion_MC.m*).

- View the inversion results by using *MASWaves_inversion_MC_plot.m* (examples are provided in Figures 2 and 3).

```
% Number of velocity reversals
N_reversals = 0; % Normally dispersive analysis

% Search-control parameters
b_S = 5;
b_h = 10;
N_max = 1000;
e_max = 0;

% Run inversion
[store_all,elapsedTime,store_accepted] = MASWaves_inversion_MC...
    (c_test,n,n_unsat,alpha_initial,nu_unsat,beta_initial,rho,h_initial,...
    N_reversals,c_OBS,lambda_OBS,up_low_boundary,c_OBS_up,c_OBS_low,...
    b_S,b_h,N_max,e_max);


%% Display inversion results
MaxDepth = 16;

MASWaves_inversion_MC_plot(c_OBS,lambda_OBS,up_low_boundary,c_OBS_up,...
    c_OBS_low,n,store_all,store_accepted,MaxDepth);
```
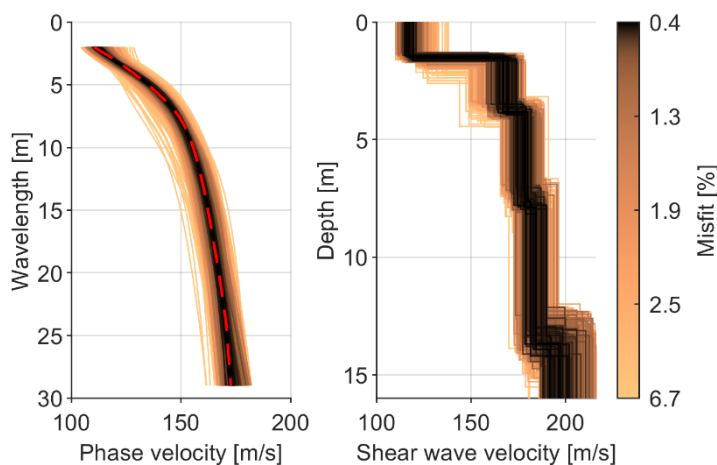

Figure 2. N_{max} = 1000 sampled shear wave velocity profiles and associated theoretical dispersion curves. The target dispersion curve (Fig. 2) is shown using a dashed red line. The shear wave velocity profiles/theoretical dispersion curves are sorted based on dispersion misfit values as indicated by the color scale.
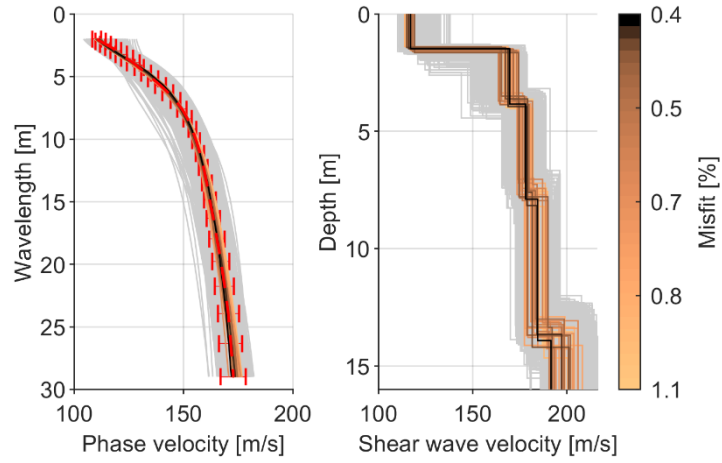
Figure 3. N_{max} = 1000 sampled shear wave velocity profiles and associated theoretical dispersion curves (gray lines). The target dispersion curve (Fig. 2) is shown using a dashed red line with horizontal error bars corresponding to plus-minus one standard deviation from the experimental curve. The shear wave velocity profiles whose theoretical dispersion curves fall within one standard deviation of the experimental data are shown using a color scale based on dispersion misfit values.

3. MASWaves Inversion

3.1 Conducting the inversion analysis (MASWaves_inversion_MC)

The function *MASWaves_inversion_MC* carries out the inversion analysis using a Monte Carlo based search procedure.

1) The program computes the theoretical (fundamental mode) dispersion curve for the stratified layer model defined by n, alpha_initial, beta_initial, rho and h_initial. The initial value of the dispersion misfit function is evaluated.
2) A Monte Carlo based search process is used in search of the shear wave velocity profile (i.e., value of beta and h for each layer) that provides the closest fit to the experimental data.
3) Inversion results are provided in the form of:
   a. All sampled profiles
   b. All sampled profiles whose theoretical dispersion curves fall within the upper/lower boundaries of the experimental curve ('accepted profiles') [optional]

Subfunctions:
MASWaves_theoretical_dispersion_curve_FDMA
MASWaves_misfit_MC

Input arguments

c_min	Minimum testing Rayleigh wave phase velocity vector [m/s]
c_max	Maximum testing Rayleigh wave phase velocity vector [m/s]
c_step	Testing Rayleigh wave phase velocity increment [m/s]
delta_c	Zero search initiation parameter [m/s]
	At wave number k_i the zero search is initiated at a phase velocity of max{c_(i-1)-delta_c , c_min}, where c_(i-1) is the theoretical Rayleigh wave phase velocity value at wave number k_(i-1)
n	Number of finite thickness layers
n_unsat	Number of unsaturated soil layers (n_unsat = 0 for a fully saturated soil profile) (n_unsat = n+1 for a fully unsaturated soil profile)
alpha_initial	Initial estimate of compressional wave velocity [m/s] (array of length n+1)
nu_unsat	Poisson's ratio
beta_initial	Initial estimate of shear wave velocity [m/s] (array of length n+1)
rho	Mass density vector [kg/m³] (array of length n+1)
h_initial	Initial estimate of layer thicknesses [m] (array of length n)
N_reversals	Number of soil layers where velocity reversals are permitted (N_reversals = 0 for a normally dispersive profile).

Experimental fundamental mode dispersion curve	
c_OBS	Phase velocity [m/s]
lambda_OBS	Wavelength [m]
up_low_boundary	- 'yes': Upper/lower boundaries for the experimental dispersion curve are available. - 'no': Upper/lower boundaries for the experimental dispersion curve are not available.
c_OBS_up	Phase velocity, upper bound curve [m/s] (can be assigned as 'nan' or [] if up_low_boundary = 'no').
c_OBS_low	Phase velocity, lower bound curve [m/s] (can be assigned as 'nan' or [] if up_low_boundary = 'no').
Search-control parameters	
b_S	Shear wave velocity search-control parameter
b_h	Layer thickness search-control parameter
N_max	Maximum number of iterations
e_max	Maximum misfit (optional stopping criterion for MC search)

Output arguments

store_all	All sampled profiles (cell array)
	   For iteration no. i
	   store_all{1,i}: Shear wave velocity vector [m/s]
	   store_all{2,i}: Layer thickness vector [m]
	   store_all{3,i}: Compressional wave velocity vector [m/s]
	   store_all{4,i}: Rayleigh wave velocity vector [m/s]
	   store_all{5,i}: Wavelength vector [m]
	   store_all{6,i}: Dispersion misfit value [%]

elapsedTime	Elapsed time (MC simulations)

store_accepted	If up_low_boundary = 'yes'
	 - Sampled profiles whose DC are within the upper/lower boundaries of the experimental data (cell array)
	   For profile no. j
	   store_accepted{1,j}: Shear wave velocity vector [m/s]
	   store_accepted{2,j}: Layer thickness vector [m]
	   store_accepted{3,j}: Compressional wave velocity vector [m/s]
	   store_accepted{4,j}: Rayleigh wave velocity vector [m/s]
	   store_accepted{5,j}: Wavelength vector [m]
	   store_accepted{6,j}: Dispersion misfit value [%]
	If up_low_boundary = 'no'
	 - store_accepted is returned as NaN

3.2 Computing theoretical dispersion curves (MASWaves_theoretical_dispersion_curve_FDMA)

The function *MASWaves_theoretical_dispersion_curve* computes the theoretical fundamental mode dispersion curve for the stratified soil model defined by n, alpha, beta, rho and h at wavelengths lambda.

Subfunctions:
MASWaves_FDMA

Input arguments

c_min	Minimum testing Rayleigh wave phase velocity vector [m/s]
c_max	Maximum testing Rayleigh wave phase velocity vector [m/s]
c_step	Testing Rayleigh wave phase velocity increment [m/s]
delta_c	Zero search initiation parameter [m/s]
	At wave number k_i the zero search is initiated at a phase velocity of max{c_(i-1)-delta_c , c_min}, where c_(i-1) is the theoretical Rayleigh wave phase velocity value at wave number k_(i-1)
lambda	Wavelength vector [m]
n	Number of finite thickness layers
alpha	Compressional wave velocity vector [m/s] (array of length n+1)
beta	Shear wave velocity vector [m/s] (array of length n+1)
rho	Mass density vector [kg/m^3] (array of length n+1)
h	Layer thickness vector [m] (array of length n)

Output arguments

c_t	Rayleigh wave phase velocity vector (theoretical dispersion curve) [m/s]
lambda_t	Rayleigh wave wavelength vector (theoretical dispersion curve) [m]

3.2.1 MASWaves_FDMA

The function *MASWaves_FDMA* computes the value of the Rayleigh wave dispersion function F for the ordered couple (c,k). Computations are conducted using the fast delta matrix algorithm (FDMA).[3] The stratified soil model is described in terms of shear wave velocity, compressional wave velocity, mass density and layer thicknesses.

Subfunctions:
MASWaves_FDMA_recursion

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[3] Buchen, P.W. & Ben-Hador, R. (1996). Free-mode surface-wave computations. *Geophysical Journal International*, 124(3), 869–887. doi:10.1111/j.1365-246X.1996.tb05642.x.

Input arguments

c	Rayleigh wave phase velocity [m/s]
k	Wave number
n	Number of finite thickness layers
alpha	Compressional wave velocity vector [m/s] (array of length n+1)
beta	Shear wave velocity vector [m/s] (array of length n+1)
rho	Mass density vector [kg/m^3] (array of length n+1)
h	Layer thickness vector [m] (array of length n)

Output arguments

F	F(c,k). Dispersion function value for the ordered couple (c,k).

3.2.2 MASWaves_FDMA_recursion

The function *MASWaves_FDMA_recursion* conductes the layer recursion of the fast delta matrix algorithm.

Subfunctions:
(none)

Input arguments

c	Rayleigh wave phase velocity [m/s]
k	Wave number
alpha	Compressional wave velocity of layer i [m/s]
beta1	Shear wave velocity of layer i [m/s]
beta2	Shear wave velocity of layer (i+1) [m/s]
rho1	Mass density of layer i [kg/m^3]
rho2	Mass density of layer (i+1) [kg/m^3]
h	Thickness of layer i [m]
X	Recursion vector

Output arguments

X	Recursion vector

3.3 Evaluating the misfit between theoretical/experimental curves (MASWaves_misfit_MC)

The function *MASWaves_misfit_MC* is used to evaluate the misfit between the theoretical and the experimental fundamental mode dispersion curves. The theoretical and experimental curves must be evaluated at the same wavelengths lambda.

Subfunctions:
(none)

Input arguments

c_t	Rayleigh wave phase velocity vector (theoretical dispersion curve) [m/s]
c_OBS	Rayleigh wave phase velocity vector (experimental/observed dispersion curve) [m/s]

Output arguments

e	Dispersion misfit [%]

3.4 Visualizing the inversion results (MASWaves_inversion_MC_plot)

The function *MASWaves_inversion_MC_plot* displays the inversion results. The simulated profiles are visualized as
1) Sampled Vs profiles/dispersion curves presented using a color scale defined based on dispersion misfit values
2) [Optional] Accepted Vs profiles/dispersion curves presented using a color scale defined based on dispersion misfit values. Remaining trial profiles shown in gray.

Subfunctions:
(None)

Input arguments

Experimental fundamental mode dispersion curve	
c_OBS	Phase velocity [m/s]
lambda_OBS	Wavelength [m]
up_low_boundary	- 'yes': Upper/lower boundaries for the experimental dispersion curve are available. - 'no': Upper/lower boundaries for the experimental dispersion curve are not available.
c_OBS_up	Phase velocity, upper bound curve [m/s] (can be assigned as 'nan' or [] if up_low_boundary = 'no').

c_OBS_low	Phase velocity, lower bound curve [m/s] (can be assigned as 'nan' or [] if up_low_boundary = 'no').
Inversion results	
n	Number of finite thickness layers

store_all	All sampled profiles (cell array)
	  For iteration no. i
	  store_all{1,i}: Shear wave velocity vector [m/s]
	  store_all{2,i}: Layer thickness vector [m]
	  store_all{3,i}: Compressional wave velocity vector [m/s]
	  store_all{4,i}: Rayleigh wave velocity vector [m/s]
	  store_all{5,i}: Wavelength vector [m]
	  store_all{6,i}: Dispersion misfit value [%]

store_accepted	Required if up_low_boundary = 'yes'
	 - Sampled profiles whose DC are within the upper/lower boundaries of the experimental data (cell array)
	  For profile no. j
	    store_accepted{1,j}: Shear wave velocity vector [m/s]
	    store_accepted{2,j}: Layer thickness vector [m]
	    store_accepted{3,j}: Compressional wave velocity vector [m/s]
	    store_accepted{4,j}: Rayleigh wave velocity vector [m/s]
	    store_accepted{5,j}: Wavelength vector [m]
	    store_accepted{6,j}: Dispersion misfit value [%]
	(can be assigned as 'nan' or [] if up_low_boundary = 'no').

MaxDepth	Maximum depth for simulated shear wave velocity profiles [m]